MIND MAP: LEARNING MADE SIMPLE CHAPTER - 4

Determinants

Minor of an element a_{ij} in a determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and is denoted by M_{ii} . If M_{ii} is the minor of a_{ii} and cofactor of a_{ii} is A_{ii} given by $A_{ii} = (-1)^{i+j} M_{ii}$.

- •If $A_{3\times 3}$ is a matrix, then $|A| = a_{11}$. $A_{11} + a_{12}$. $A_{12} + a_{13}$. A_{13} .
- •If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For e.g., $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{33} = 0$.

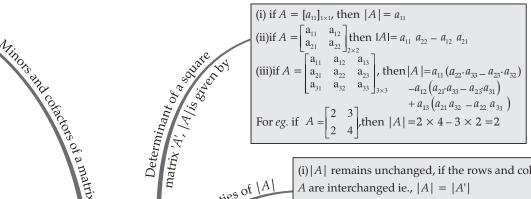
e.g., if
$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$
, then $M_{11} = 4$ and $A_{11} = (-1)^{1+1} = 4 = 4$.

if
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$
, then adj. $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

,where A_{ii} is the cofactor of a_{ii}.

- A(adj.A) = (adj.A).A = |A| = I, A square matrix
- if |A| = 0, then A is singular. Otherwise, A is non-singular.
- if AB = BA = I, where B is a square matrix, then B is called the inverse of A, $A^{-1}=B$ or $B^{-1}=A$, $(A^{-1})^{-1} = A$.

Inverse of a square matrix exists if A is non-singular i.e. $|A| \neq 0$, and is given by $A^{-1} = \frac{1}{|A|} (adj.A)$



properties of |A|

- (i) |A| remains unchanged, if the rows and columns of A are interchanged ie., |A| = |A'|
- (ii)if any two rows (or columns) of A are interchanged, then the sign of |A| changes.
- (iii)if any two rows (or columns) of A are identical, then |A| = 0
- (iv)if each element of a row (or a column) of *A* is multiplied by B (const.), then |A| gets multiplied by B. (v) if $A = \begin{bmatrix} a_{ij} \end{bmatrix}_{x = x}$ then $|k.A| = k^3 |A|$.
- (vi) if elements of a row or a column in a determinant |A| can be expressed as sum of two or more elements, then |A| can be expressed as |B| + |C|.
- (vii) if $R_i \rightarrow R_i + kRj$ or $C_i = C_i + kCj$ in |A|, then the value of |A| remains same

• if
$$a_1x + b_1y + c_1z = d_1$$
, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$ then we can write $AX = B$,

Adding and inverse

Applications of determine of determinations of d

where
$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$
 $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

- Unique solution of AX = B is $X = A^{1}B$, $|A| \neq 0$.
- AX = B is consistent or inconsistent according as the solution exists or not.
- For a square matrix A in AX=B, if
 - (i) $|A| \neq 0$ then there exists unique solution.
- (ii) |A| = 0 and (adj. A) $B \neq 0$, then no solution.
- (iii) if |A| = 0 and (adj.A).B = 0 then system may or may not be consistent.

if
$$(x_1, y_1)$$
, (x_2, y_2) and (x_3, y_3) $\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

For eg: if (1, 2), (3, 4) and (-2, 5) are the vertices, then area of the triangle is

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ -2 & 5 & 1 \end{vmatrix} = 1(4-5)-2(3+2)+1(15+8)=12 \text{ sq.units}$$

we take positive value of the determinant.



